

On a possible breaking of global N=2 supersymmetry in non-linear σ models on compact Kähler target spaces

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Abstract

We analyse with the algebraic, regularisation independent, cohomological B.R.S. methods, the renormalisability of torsionless N=2 supersymmetric non-linear σ models built on compact Kähler spaces. Surprisingly enough with respect to the common wisdom, we obtain an anomaly candidate, when the Hodge number $h^{3,0}$ of the target space manifold is different from zero : this occurs in particular in the Calabi-Yau case. On the contrary, in the compact homogeneous Kähler case, the anomaly candidate disappears.

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1 Introduction

Supersymmetric non-linear σ models in two space time dimensions have been considered for many years to describe the vacuum state of superstrings [1][2]. In particular Calabi-Yau spaces, *i.e.* 6 dimensional compact Kähler Ricci-flat manifolds [3], appear as good candidates in the compactification of the 10 dimensional superstring to 4 dimensional flat Minkowski space ; the conformal invariance of the 2.d, $N = 2$ supersymmetric non-linear σ model (the fields of which are coordinates on this compact manifold) is expected to hold to all orders of perturbation theory [4].

However explicit calculations to 4 or 5 loops [5] and, afterwards, general arguments [6] show that the β functions may not vanish. But, as argued in a recent review [7], at least two problems obscure these analyses : first, the fact that the quantum theory is not sufficiently constrained by the Kähler Ricci-flatness requirement ; second, the use of “dimensional reduction” [8] or of harmonic superspace formalism [9] ¹ in actual explicit calculations and general arguments. Then, we prefer to analyse these models using the B.R.S., algebraic, regularisation free cohomological methods.

So we adress ourselves to the question of the all-order renormalisability of $N = 2$ supersymmetric non linear σ models in two space time dimensions.

Our main result is that, surprisingly enough with respect to the common wisdom ², there exists a possible anomaly for global supersymmetry in 2 space-time dimensions, at least for torsionless compact Kähler Ricci-flat manifolds (*i.e.* special $N=2$ supersymmetric models)[13].

2 The classical theory and the Slavnov operator

We use $N = 1$ superfields ³ $\phi^i(x^+, x^-, \theta^+, \theta^-)$ and, in the absence of torsion, the most general $N = 1$ invariant action is :

$$S^{inv.} = \int d^2x d^2\theta g_{ij}[\phi] D_+ \phi^i D_- \phi^j \quad (1)$$

where the supersymmetric covariant derivatives

$$D_{\pm} = \frac{\partial}{\partial \theta^{\pm}} + i\theta^{\pm} \frac{\partial}{\partial x^{\pm}}$$

satisfy

$$\{D_+, D_-\} = 0 \quad , \quad D_{\pm}^2 = i \frac{\partial}{\partial x^{\pm}} \equiv i\partial_{\pm} \quad .$$

¹ The regularisation through dimensional reduction suffers from algebraic inconsistencies and the quantization in harmonic superspace does not rely on firm basis, due to the presence of non-local singularities (in the harmonic superspace) [10].

² Notice also that recent works of Brandt [11] and Dixon [12] show the existence of new non-trivial cohomologies in supersymmetric theories.

³ The quantization with $N = 1$ superfields was put on firm basis by Piguet and Rouet [14] who proved in particular the Quantum Action Principle in that context. Moreover, in [15]a) we show the renormalisability of $N=1$ supersymmetric non-linear σ models using component fields : this justifies the use of $N=1$ superfields for the present analysis of extended supersymmetry.

Notice also that we use light-cone coordinates.

The tensor $g_{ij}[\phi]$ is interpreted as a metric tensor on a Riemannian manifold \mathcal{M} ⁴. As is by now well known [18], $N = 2$ supersymmetry needs \mathcal{M} to be a $2n$ dimensional Kähler manifold. The second supersymmetry transformation writes :

$$\delta\phi^i = J_j^i[\phi](\epsilon^+ D_+ \phi^j + \epsilon^- D_- \phi^j) . \quad (2)$$

where J_j^i is a covariantly constant integrable complex structure. In the B.R.S. approach, the supersymmetry parameters ϵ^\pm are promoted to constant, commuting Faddeev-Popov parameters d^\pm and an anticommuting classical source η_i for the non-linear field transformation (2) is introduced in the classical action⁵:

$$\Gamma^{class.} = \int d^2x d^2\theta \left\{ g_{ij}[\phi] D_+ \phi^i D_- \phi^j + \eta_i J_j^i[\phi] (d^+ D_+ \phi^j + d^- D_- \phi^j) \right\} . \quad (3)$$

For simplicity, no mass term is added here as we are only interested in U.V. properties. The non linear Slavnov operator is defined by

$$S\Gamma \equiv \int d^2x d^2\theta \frac{\delta\Gamma}{\delta\eta_i(x, \theta)} \frac{\delta\Gamma}{\delta\phi^i(x, \theta)}$$

and we find

$$S\Gamma^{class.} = (d^+)^2 \int d^2x d^2\theta \eta_k i \partial_+ \phi^k + (d^-)^2 \int d^2x d^2\theta \eta_k i \partial_- \phi^k$$

in accordance with the supersymmetry algebra.

As is by now well known (for example see [7] or [16]), in the absence of a consistent regularisation that respects all the symmetries of the theory, the quantum analysis directly depends on the cohomology of the nilpotent linearized Slavnov operator :

$$\begin{aligned} S_L &= \int d^2x d^2\theta \left[\frac{\delta\Gamma^{class.}}{\delta\eta_i(x, \theta)} \frac{\delta}{\delta\phi^i(x, \theta)} + \frac{\delta\Gamma^{class.}}{\delta\phi^i(x, \theta)} \frac{\delta}{\delta\eta_i(x, \theta)} \right] \\ S_L^2 &= 0 \end{aligned} \quad (4)$$

in the Faddeev-Popov charge +1 sector [absence of anomalies for the $N = 2$ supersymmetry] and charge 0 sector [number of physical parameters and stability of the classical action through renormalization]. Notice that the Slavnov coboundaries $S_L \int d^2x d^2\theta \eta_i W^i[\phi]$ correspond to field and source reparametrisations :

$$\phi^i \rightarrow \phi^i + \lambda W^i[\phi] \quad , \quad \eta_i \rightarrow \eta_i - \lambda \eta_k W_{,i}^k[\phi] , \quad (5)$$

where $W^i[\phi]$ is an arbitrary function of the fields $\phi(x, \theta)$ and a comma indicates a derivative with respect to the field ϕ^i .

⁴ Here, contrarily to our previous work where the manifold \mathcal{M} was supposed to be an homogeneous space [16], we consider renormalisability “à la Friedan” [17], *i.e.* in the space of metrics, and analyse only the possibility of maintaining to all orders the $N=2$ supersymmetry. As explained in [7], in order to define unambiguously the classical action, one should add extra properties.

⁵ In the absence of torsion, there is a parity invariance

$$+ \rightarrow -, d^2x \rightarrow d^2x, d^2\theta \rightarrow -d^2\theta, \phi^i \rightarrow \phi^i, \eta_i \rightarrow -\eta_i .$$

Moreover, the canonical dimensions of $[d^2x d^2\theta]$, $[\phi^i]$, $[d^\pm]$, $[D_\pm]$, $[\eta_i]$ are -1, 0, -1/2, +1/2, +1 respectively and the Faddeev-Popov assignments +1 for d^\pm , -1 for η_i , 0 for the other quantities.

Due to the highly non-linear character of S_L (equ.(4)), it is convenient to use a “filtration” ([19],[16]) with respect to the number of fields $\phi^i(x, \theta)$ and their derivatives. As it does not change this number, the nilpotent lowest order part of S_L , S_L^0 , will play a special role :

$$\begin{aligned} S_L &= S_L^0 + S_L^1 + S_L^2 + \dots \equiv S_L^0 + S_L^r, \quad (S_L^0)^2 = (S_L^r)^2 + S_L^0 S_L^r + S_L^r S_L^0 = 0 \\ S_L^0 &= \int d^2 x d^2 \theta J_j^i(0) \left\{ (d^+ D_+ \phi^j + d^- D_- \phi^j) \frac{\delta}{\delta \phi^i} + (d^+ D_+ \eta_i + d^- D_- \eta_i) \frac{\delta}{\delta \eta_j} \right\}. \end{aligned} \quad (6)$$

As explained in refs.[16] and [15]a), when S_L^0 has no cohomology in the Faddeev-Popov positively charged sectors, the cohomology of the complete S_L operator in the Faddeev-Popov sectors of charge 0 and +1 is isomorphic to a subspace of the one of S_L^0 in the same sectors.

Then, we first analyse the possible cohomology of the Slavnov operator in the anomaly sector : indeed, if there exists a non-trivial cohomology, this is the death of the theory (for a complete analysis we refer to [13] and [15]b)).

3 A candidate for an anomaly

According to the spectral sequence method, we first analyse the possible cohomology of S_L^0 in the anomaly sector.

3.1 S_L^0 cohomology

The most general dimension zero integrated local polynomial in the Faddeev-Popov parameters, fields and their derivatives, of Faddeev-Popov +1 charge writes :

$$\begin{aligned} \Delta_{[+1]} &= \int d^2 x d^2 \theta \{ t^{[ijk]} (d^+)^2 (d^-)^2 \eta_i \eta_j \eta_k \\ &+ d^+ d^- [\eta_i \eta_j t_{1n}^{[ij]} (d^+ D_+ \phi^n - d^- D_- \phi^n) + \eta_n t_{2[ij]}^n D_+ \phi^i D_- \phi^j] \\ &+ d^+ d^- \eta_i s_1^{(ij)} (d^+ D_+ \eta_j - d^- D_- \eta_j) \\ &+ (d^+)^2 (\eta_n t_{3[ij]}^n D_+ \phi^i D_+ \phi^j + D_+ \eta_i D_+ \phi^j t_{4j}^i) \\ &+ (d^-)^2 (\eta_n t_{3[ij]}^n D_- \phi^i D_- \phi^j + D_- \eta_i D_- \phi^j t_{4j}^i) \\ &+ d^+ (\tilde{t}_{[ij]n} D_+ \phi^i D_+ \phi^j D_- \phi^n + s_{2(ij)} D_- D_+ \phi^i D_+ \phi^j) \\ &- d^- (\tilde{t}_{[ij]n} D_- \phi^i D_- \phi^j D_+ \phi^n + s_{2(ij)} D_+ D_- \phi^i D_- \phi^j) \} \end{aligned} \quad (7)$$

where, due to the anticommuting properties of η_i and $D_\pm \phi^i$ and to the integration by parts freedom, the tensors $t^{[ijk]}$, $t_{1n}^{[ij]}$, $t_{2[ij]}^n$, $t_{3[ij]}^n$, $\tilde{t}_{[ij]n}$ are antisymmetric in i, j, k, and $s_1^{(ij)}$, $s_{2(ij)}$ symmetric in i, j.

The analysis of the cocycle condition $S_L^0 \Delta_{[+1]} = 0$ leads to :

$$\begin{aligned} \Delta_{[+1]} &= \Delta_{[+1]}^{an.} [t^{[ijk]}(\phi)] + S_L^0 \Delta_{[0]} [\text{arbitrary } t_{ij}(\phi) \text{ and } U_j^i(\phi)] \\ \text{where } \Delta_{[0]} &= \int d^2 x d^2 \theta \{ t_{ij}[\phi] D_+ \phi^i D_- \phi^j + \eta_i U_j^i[\phi] (d^+ D_+ \phi^j + d^- D_- \phi^j) \} \end{aligned} \quad (8)$$

and where the antisymmetric tensor $t^{[ijk]}(\phi)$ which occurs in the anomalous part

$$\Delta_{[+1]}^{an.} = \int d^2 x d^2 \theta t^{[ijk]}(\phi) (d^+)^2 (d^-)^2 \eta_i \eta_j \eta_k \quad (9)$$

is constrained by :

$$\begin{aligned} a) \quad & J_n^i(0)t^{[njk]} \quad \text{is } i, j, k \text{ antisymmetric,} \\ b) \quad & J_n^i(0)t_{,m}^{[njk]} = J_m^n(0)t_{,n}^{[ijk]} \end{aligned} \quad (10)$$

These conditions, when expressed in a coordinate system adapted to the complex structure $J_j^i[\phi]$ ($i \equiv (\alpha, \bar{\alpha})$: $J_\beta^\alpha = i\delta_\beta^\alpha$, $J_{\bar{\beta}}^{\bar{\alpha}} = -i\delta_{\bar{\beta}}^{\bar{\alpha}}$, $J_\beta^{\bar{\alpha}} = J_{\bar{\beta}}^\alpha = 0$) mean that the tensor $t^{[ijk]}$ is a pure contravariant skewsymmetric analytic tensor (*i.e.* $t^{[\alpha\beta\gamma]} = t^{[\alpha\beta\gamma]}(\phi^\delta)$, $t^{[\bar{\alpha}\bar{\beta}\bar{\gamma}]} = t^{[\bar{\alpha}\bar{\beta}\bar{\gamma}]}(\bar{\phi}^{\bar{\delta}})$), the other components vanish). In particular, due to the vanishing of $t^{[\alpha\beta\bar{\gamma}]}$, such tensor cannot be a candidate for a torsion tensor on a Kähler manifold [20].

Consider the covariant tensor

$$t_{[\alpha\beta\gamma]} = g_{\alpha\bar{\alpha}}g_{\beta\bar{\beta}}g_{\gamma\bar{\gamma}}t^{[\bar{\alpha}\bar{\beta}\bar{\gamma}]} .$$

It satisfies $\nabla_\delta t_{[\alpha\beta\gamma]} = 0$. Then the (3-0) form

$$\omega' = \frac{1}{3!} t_{[\alpha\beta\gamma]} d\phi^\alpha \wedge d\phi^\beta \wedge d\phi^\gamma$$

which satisfies $d'\omega' = 0$, may be shown to be harmonic as \mathcal{M} is a compact manifold ⁶ ([21],[15]b)). It is known that the number of such forms is given by the Hodge number $h^{3,0}$: then this number determines an upper bound for the dimension of the cohomology space of S_L in the anomaly sector.

As a first result, this proves that if the manifold \mathcal{M} has a complex dimension smaller than 3, there is no anomaly candidate. Another special case is the compact Kähler homogeneous one ($N=2$ supersymmetric extension of our previous work on the bosonic case [16]) : in such a case the Ricci tensor is positive definite [22] which forbids ([21],[15]b)) the existence of such analytic tensor $t^{[\alpha\beta\gamma]}(\phi^\delta)$. As a consequence, the cohomology of S_L^0 - and then of S_L - vanishes in the anomaly sector (for details, see ref.[15]b)).

On the contrary, when $h^{3,0} \neq 0$, we have a candidate for an anomaly. We shall now discuss the possible non-trivial cohomology of the complete $S_L \equiv S_L^0 + S_L^r$ operator, still in the Faddeev-Popov charge +1 sector.

3.2 S_L cohomology

Starting from the S_L^0 cohomology (9), we were able to construct the S_L cohomology in the same Faddeev-Popov sector ⁷ ([15]b)) :

$$\Delta_{[+1]}^{an.} = \int d^2x d^2\theta t^{[ijk]}[\phi] \{ (d^+)^2 (d^-)^2 \eta_i \eta_j \eta_k$$

⁶ In this Kählerian case, one firstly obtains from $\nabla_\delta t_{[\alpha\beta\gamma]} = 0$, $\Delta t_{[\alpha\beta\gamma]} = g^{\delta\bar{\delta}} \nabla_\delta \nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]} - [R_\alpha^\delta t_{[\delta\beta\gamma]} + \text{perms.}]$; on another hand, the Ricci identity gives $g^{\delta\bar{\delta}} \nabla_\delta \nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]} = -[R_\alpha^\delta t_{[\delta\beta\gamma]} + \text{perms.}]$. So $\Delta t_{[\alpha\beta\gamma]} = 2g^{\delta\bar{\delta}} \nabla_\delta \nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]}$. Now, the manifold being compact, one may compute :

$$\begin{aligned} & (\delta\omega', d\omega') + (d\omega', \delta\omega') = (\omega', (d\delta + \delta d)\omega') = (\omega', \Delta\omega') = \\ & = \int_{\mathcal{M}} d\sigma 2t^{[\alpha\beta\gamma]} g^{\delta\bar{\delta}} \nabla_\delta \nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]} = \int_{\mathcal{M}} d\sigma 2g^{\delta\bar{\delta}} \{ \nabla_\delta \nabla_{\bar{\delta}} (t^{[\alpha\beta\gamma]} t_{[\alpha\beta\gamma]}) - \nabla_\delta t^{[\alpha\beta\gamma]} \nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]} \} = 0 - 2(d\omega', d\omega') \\ & \Rightarrow (\delta\omega', \delta\omega') + 3(d\omega', d\omega') = 0 \Rightarrow \delta\omega' = d\omega' = \Delta\omega' = 0 . \quad Q.E.D. \end{aligned}$$

⁷ In this compact case, due to the harmonicity of the (3,0) form ω' , one gets from $d\omega' = d''\omega' = 0$ the vanishing of $\nabla_{\bar{\delta}} t_{[\alpha\beta\gamma]}$, which means that t_{ijk} is a pure covariant skewsymmetric analytic tensor.

$$\begin{aligned}
& - \frac{3}{2} d^+ d^- (\eta_i \eta_j J_{kn} (d^+ D_+ \phi^n - d^- D_- \phi^n) + 2 \eta_i J_{jn} J_{km} D_+ \phi^n D_- \phi^m) \\
& + \frac{3}{4} J_{in} J_{jm} J_{kl} (d^+ D_+ \phi^n D_+ \phi^m D_- \phi^l - d^- D_- \phi^n D_- \phi^m D_+ \phi^l) \}
\end{aligned} \tag{11}$$

As a consequence, if at a given perturbative order this anomaly appears with a non zero coefficient

$$S_L \Gamma|_{p^{th} order} = a(\hbar)^p \Delta_{[+1]}^{an.}, \quad a \neq 0$$

the $N = 2$ supersymmetry is broken as $\Delta_{[+1]}^{an.}$ cannot be reabsorbed (being a cohomology element, it is not a $S_L \tilde{\Delta}_{[0]}$) and, *a priori*, we are no longer able to analyse the structure of the U.V. divergences at the next perturbative order, which is the death of the theory. When the target space is a Ricci-flat one, such tensor may exist : Calabi-Yau manifolds (3 complex dimensional case) where $h^{3,0} = 1$ [3] are interesting examples⁸ due to their possible relevance for superstring theories. Of course, as no explicit metric is at hand, one can hardly compute the anomaly coefficient.

4 Concluding remarks

We have analysed the cohomology of the B.R.S. operator associated to $N = 2$ supersymmetry in a $N = 1$ superfield formalism. We have found an anomaly candidate for torsionless models built on compact Kähler target spaces with a non vanishing Hodge number $h^{3,0}$. This anomaly in global extended supersymmetry is a surprise with respect to common wisdom [24] (but see other unexpected cohomologies in supersymmetric theories, in the recent works of Brandt [11] and Dixon [12]).

Of course, our analysis casts some doubts on the validity of the previous claims on U.V. properties of $N=2$ supersymmetric non linear σ models : there, the possible occurrence at 4-loops order of (infinite) counterterms non-vanishing on-shell, even for Kähler Ricci-flat manifolds, did not “disturb” the complex structure ; on the other hand, we have found a possible “instability” of the second supersymmetry, which confirms that there are some difficulties in the regularisation of supersymmetry by dimensional reduction assumed as well in explicit perturbative calculations [5] than in finiteness “proofs” [4] or higher order counterterms analysis [6]. We would like to emphasize the difference between Faddeev-Popov 0 charge cohomology which describes the stability of the classical action against radiative corrections (the usual “infinite” counterterms) and which offers no surprise(see [15]b) for a detailed analysis), and the anomaly sector which describes the “stability” of the symmetry (the finite renormalisations which are needed, in presence of a regularisation that does not respect the symmetries of the theory, to restore the Ward identities) : of course, when at a given perturbative order the Slavnov (or Ward) identities are spoiled, at the next order, the analysis of the structure of the divergences is no longer under control. In particular, the Calabi-Yau uniqueness theorem for the metric [25] invoked in some finiteness “proofs” [4] supposes that one stays in the same cohomology class for the Kähler form, a fact which is not certain in the absence of a regularisation that respects the $N=2$ supersymmetry (the possible anomaly we found expresses the impossibility to find a regularisation that respects all the symmetries of these theories).

We emphasize the fact that the present work relies heavily on perturbative analysis, especially through the use of the Quantum Action Principle in order to analyse the possible breakings of Slavnov identities. It may well happen that the coefficient of the anomaly candidate

⁸ As $\det \|g\| = 1$, a representative of $t^{[\alpha\beta\gamma]}$ is the constant skewsymmetric tensor $\epsilon^{[\alpha\beta\gamma]}$ (with $\epsilon^{123} = +1$).

vanishes at any finite order of perturbation theory⁹, the existence of a non-trivial supersymmetry cohomology leaving open the interesting possibility of a non-perturbative breaking of $N=2$ supersymmetry.

Finally, when the manifold is a compact homogeneous Kähler space, the anomaly candidate disappears as expected. Moreover, we have also been able to prove that, if one enforces $N=4$ supersymmetry (HyperKähler manifolds), the anomaly vanishes ([15]b)).

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⁹ An argument based on the universality of the coefficient at any finite order of perturbation theory and its vanishing for a special class of Calabi-Yau manifolds corresponding to orbifolds of tori has been given to the author by the referee of [13].

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